# Modified Sequence Algorithm for Computation of Optimum sequencing for Production scheduling 

Naga Sai Ram.G, V.S. Haswanth.G, Durga Rajesh.K


#### Abstract

This paper focuses on the computation of the optimal sequence of the 2-Machines and $n$-Jobs. Conventional Johnson algorithm is long and often required time to compute them. Scaling such algorithm for m-machines $n$-Jobs is a bit complicated. The application of modified sequence algorithm to the $n$ Jobs and $m$ Machines is also made, which is the checking criteria for the $n \times 3, n \times 5$ and $n \times m$ scheduling problems change in the conventional algorithm by verifying the jobs value weather it is small and larger and appending them in priority sequence can help to solve such problems.


Index Terms- Optimum sequence, m-machines $n$-Jobs, Seheduling problem.

## 1 Introduction

Scheduling algorithms are used to find out the optimum sequence of the machines to be utilized for the optimum production. The application of the algorithm depends upon the number of jobs and machines in the problem. The basic possible case is $n$ jobs 2 Machines where 2 machines are assigned by $n$ jobs. Johnson algorithm is one such solving procedure for attaining the optimum solution of those problems. But this algorithm cannot be applied to the $n$ jobs and $m$ machines. The computed time for the Johnson algorithm depended on the number of searches taken to find the smallest time of in the job. In the computation terms this is represented by the $\mathrm{O}\left(\mathrm{N}^{2}\right)$. Which take time to find the optimum sequence for larger scaled problems. In order to decrease the time and the ease of computations of the sequence a small change has been made and rest of the algorithm is made to be same.

## 2 LITERATURE REVIRE

The first approach for the scheduling the two and three jobs production problem have been made by the Johnson S.M in $1954{ }^{[1]}$. Solving the n jobs and m machines procedure have been given by the Herbert Campbell and Richard A.Duke without use of computer was published in 1970 ${ }^{[2]}$.

## 3 METHODOLOGY

The basic work flow of this algorithm is to sort out the least time possible for the machines according to the jobs given. Which is the similar step in the normal Johnson algorithm. But the way of selecting the job is different in this method by selecting the smallest possible machine time in the machine cells and depending upon the positions of the smallest possible time appending the job with respect to the smallest entry. When it comes to the large scale problems it is easy to check the simple logic

[^0]and decide the optimum sequence of the jobs. By using the divide and rule procedure the modified Johnson algorithm can be scaled up to $m$ machines by using the resultant sequence is compared to the next machines and the procedure continues until the total number of the columns are converged. This algorithm can only be applied to the problem based on the time criteria.
Checking the optimal solution for the sequencing problem or deciding the optimal sequence based on the process cannot be the deciding factor, it is needed to check the all the possible cases say 2 n where n is the total number of the jobs. This adaptation is not possible for the higher orders of $n$, because there lie only one optimum solutions in ${ }^{n} C_{1}$ where n is the number of the jobs.
sometimes chances of failing happen when made with conventional algorithm and alternate procedures needed to verify this cases and they may find much more optimal solution. So Modified Sequencing algorithm will be the checking criteria.

## 3.A) MODIFIED SEQUENCE ALGORITHM

The modified sequence algorithms have the following operational steps:

1. Calculate number of columns which is $m$.
2. Calculate convergence limit which is $m-1$.
3. For $i=0$ compare $i+1, i+2$ columns for minimum row value.
4. If the minimum row value is in $i+1$ columns place the job value and respective minimum row value at front.
5. If the minimum row value is in $i+2$ place the job value and the respective minimum row value at end.
6. If the values of $i+1$ and $i+2$ are equal, then place the job value and respective row value at end.
7. Increment $i$.
8. Compare the newly formed list with $i+m-1$.
9. Continue step $: 3$ until convergenc limit
10. Write only the job list.
we can apply the above algorithm only to the 2 Machines and $n$ jobs which return the optimum sequence and this algorithm can be scaled to the $n$ jobs and $m$ machines.

## 3.в) ANALYSIS OF THE ALGORITHM

The time taken to solve the $\mathrm{n} \times \mathrm{m}$ job machine sequencing problems is very high both computationally and manually, whereas the modified sequencing algorithms takes very low time in either way. To prove the computational time of the modified sequencing algorithm the following considerations are taken.
For nxm job machine sequence matrix for $T_{c}=[n+m-1] O(m)$. Number of job interchange iterations is $=n$.
Number of comparison iterations is $=\mathrm{m}-1$.
Time taken to process single line of iteration is $\mathrm{O}(\mathrm{m})$.
The total time complexity of the modified sequencing algrithm is

$$
\begin{equation*}
T_{c}=n \times O(m)+m-1 \times O(m) . \tag{1}
\end{equation*}
$$

$\qquad$

$$
T_{c}=[n+m-1] O(m) \ldots \ldots \ldots . . \text { (2) where }(m, n \geq 2)
$$

[ $n+m-1$ ] is also called as the trans variable coefficient. The for the values of above and equal to 2 the progression starts as 3,5,7. $\qquad$ .. $\mathrm{n}^{\text {th }}$ value is $\mathrm{n}+2$ so the limiting factor of this algorithm is below 2 . The modified sequencing algorithm is a linear algorithm. The advantages of this algorithm is as follows.

1. Modified sequencing algorithm is an non - destructive in nature all the values of the job machine values are not deleted or removed.
2. Modified sequencing algorithm can be scaled to any size of job machine matrix.
3. Higher optimization results can be obtained when compared to the conventional algorithms.
4. The time taken to calculate or compute the solution is lesser when compared to conventional algorithm.
5. Faster iteration convergences along with the estimation of the convergence can be made.
6. Modified sequencing algorithm is linear in nature when coded and takes lower memory.
7. Operational logic is simple in nature.

## 4 NUMERICAL EXAMPLES.

The following are the comparisons between the modified sequencing algorithm and the conventional sequencing algorithm.
Example 1: for the $4 \times 2$ job machine matrix shown below which is solved by conventional Johnson algorithm.

| Job | Machine 1 | Machine 2 |
| :---: | :---: | :---: |
| A | 3 | 4 |
| B | 5 | 2 |
| C | 2 | 3 |
| D | 6 | 2 |

Sol: check the smallest time between columns for the minimum values and placed them in front or end by the place of the value.

| Job | Machine 1 | Machine 2 |
| :---: | :---: | :---: |
| A | 3 | 4 |
| B | 5 | 2 |
| C | 2 | 3 |
| D | 6 | 2 |


| Job | Machine 1 | Machine 2 |
| :--- | :--- | :--- |
| A | 3 | 4 |
| B | 5 | 2 |
| C | 2 | 3 |
| D | 6 | 2 |

Smallest value out of all cells is 2 which is in machine 2 so placed at end.


Smallest value out of all cells is 2 which is in the machine 1 so place the job in the front


Smallest value out of all cells is 2 which is in the machine 2 so place the job in the end.

$$
\begin{array}{|l|l|l|l|}
\hline \mathrm{C} & & \mathrm{D} & \mathrm{~B} \\
\hline
\end{array}
$$

The optimum sequence is as above and the total time estimation is as follows.

| Job | Machine 1 |  | Machine 2 |  |
| :---: | :---: | :---: | :---: | :---: |
| C | 0 | 2 | 2 | 5 |
| A | 2 | 5 | 5 | 9 |
| D | 5 | 11 | 11 | 13 |
| B | 11 | 16 | 16 | 18 |

The time elapsed estimation is 18.
For the same problem application of the Modified sequencing algorithm which gives the following results.

| Job | Machine 1 | Machine 2 |
| :---: | :---: | :---: |
| A | 3 | 4 |
| B | 5 | 2 |
| C | 2 | 3 |
| D | 6 | 2 |


| Job | Machine <br> 1 | Machine <br> 2 | Minimum <br> row values | Modified job <br> sequence |
| :---: | :---: | :---: | :---: | :---: |
| A | 3 | 4 | 3 | A |
| B | 5 | 2 | 2 | C |
| C | 2 | 3 | 2 | D |
| D | 6 | 2 | 2 | B |

The optimum sequences of the jobs are as follows

$$
\begin{array}{|l|l|l|l|}
\hline \mathrm{A} & \mathrm{C} & \mathrm{D} & \mathrm{~B} \\
\hline
\end{array}
$$

## Algorithm analysis.

The value of $m$ is 2 .
The convergence limit count is $m-1$
= $2-1=1$
The number of job inter change is $n$ which is equal to 4 .
The trans variable coefficient is $n+m-1=4+1=5$ which is above 2 hence verified.
From the above table it is observed that the optimum sequence for the both the methods are same hence total justification is made and the total time estimations is as follows.

| Job | Machine 1 |  | Machine 2 |  |
| :--- | :--- | :--- | :--- | :---: |
| A | 0 | 3 | 3 | 7 |
| C | 3 | 5 | 7 | 10 |
| D | 5 | 11 | 11 | 13 |
| B | 11 | 16 | 16 | 18 |

The values obtained here is optimal than that of the Johnson algorithm hence there lies the total verification with the Johnson algorithm.
Example 2: For the $4 \times 5$ job machine matrix shown below which is solved by convectional algorithm is as follows.

|  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Jobs | Machine <br> 1 | Machine <br> 2 | Machine <br> 3 | Machine <br> 4 | Machine <br> 5 |
|  |  |  |  |  |  |
| A | 8 | 6 | 2 | 3 | 9 |
| B | 6 | 6 | 4 | 5 | 10 |
| C | 5 | 4 | 5 | 6 | 8 |
| D | 8 | 3 | 3 | 2 | 6 |

Minimum processing time of Machine $\mathrm{A} \geq$ maximum processing time of Machine (B, C, D). This case is failed because $5 \geq(6,5,6)$
Minimum processing time of Machine $\mathrm{E} \geq$ maximum processing time of Machine ( $\mathrm{B}, \mathrm{C}, \mathrm{D}$ ). this case is working because $6 \geq(6,5,6)$.
Converting the 5 Machine problem to 2 Machine problem by assuming the imaginary Machine's G, H.
Where $G=A+B+C+D$.

## $H=B+C+D+E$.

The following is the time setup for $G, H$ with job specified.

| Jobs | Machine$1$ |  | $\begin{gathered} \text { Machine } \\ 2 \end{gathered}$ |  | $\begin{gathered} \text { Machine } \\ 3 \\ \hline \end{gathered}$ |  | $\begin{gathered} \text { Machine } \\ 4 \\ \hline \end{gathered}$ |  | $\begin{gathered} \hline \text { Machine } \\ 5 \\ \hline \end{gathered}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | in | out | in | out | in | out | in | out | In | out |
| 1 | 0 | 8 | 8 | 14 | 14 | 16 | 16 | 19 | 19 | 28 |
| 3 | 8 | 13 | 14 | 19 | 18 | 23 | 23 | 29 | 29 | 37 |
| 2 | 13 | 19 | 19 | 25 | 25 | 29 | 29 | 34 | 37 | 47 |
| 4 | 19 | 27 | 27 | 30 | 30 | 33 | 34 | 36 | 47 | 53 |

for the $4 \times 5$ job machine matrix shown below which is solved

| Jobs | Machine <br> 1 | Machine <br> 2 | Machine <br> 3 | Machine <br> 4 | Machine <br> 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| A | 8 | 6 | 2 | 3 | 9 |
| B | 6 | 6 | 4 | 5 | 10 |
| C | 5 | 4 | 5 | 6 | 8 |
| D | 8 | 3 | 3 | 2 | 6 |

## Sol)

The size of the job machine matrix is $4 \times 5$.
The convergence limit is $m-1=5-1=4$
So the number of iterations are 4.
Iteration :1

| jobs | Machine | Machine | Minimum row | Modified job |
| :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | values | values |


| A | 8 | 6 | 3 | D |
| :---: | :--- | :--- | :--- | :--- |
| B | 6 | 6 | 4 | C |
| C | 5 | 4 | 6 | B |
| D | 8 | 3 | 6 | A |

Iteration :2

| Modified <br> job values | Minimum <br> row values | Machine <br> 3 | Minimum <br> row values | Modified <br> job values |
| :---: | :---: | :---: | :---: | :---: |
| D | 3 | 2 | 3 | A |
| C | 4 | 4 | 5 | B |
| B | 6 | 5 | 4 | C |
| A | 6 | 3 | 2 | D |

Iteration :3

| Modified | Minimum | Machine | Minimum | Modified |
| :---: | :---: | :---: | :---: | :---: |
| job values | row values | 4 | row values | job values |


| A | 3 | 3 | 4 | C |
| :---: | :--- | :--- | :--- | :--- |
| B | 5 | 5 | 2 | D |
| C | 4 | 6 | 5 | B |
| D | 2 | 2 | 3 | A |

Iteration :4
\(\left.$$
\begin{array}{l}\begin{array}{c}\text { Modified } \\
\text { job values }\end{array} \\
\begin{array}{c}\text { Minimum } \\
\text { row values }\end{array} \\
\hline \text { Machine } \\
5\end{array}
$$ $$
\begin{array}{c}\text { Minimum } \\
\text { row values }\end{array}
$$ \begin{array}{c}Modified <br>

job values\end{array}\right]\)| C | 4 | 9 | 4 |
| :---: | :---: | :---: | :---: |
| D | 2 | 10 | 2 |
| B | 5 | 8 | 5 |
| A | 3 | 6 | 3 |

Total elapsed time of the machines are as follows.

| Job | Machine |  |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | 1 |  | Machine |  |  |  |  |  |  |  |  |
| 2 |  | Machine |  |  |  | Machine | Machine |  |  |  |  |
|  | in | out | in | out | in | out | in | out | In | out |  |
|  | C | 0 | 5 | 5 | 9 | 9 | 14 | 14 | 20 | 20 | 28 |
| D | 5 | 13 | 13 | 16 | 16 | 19 | 19 | 22 | 28 | 34 |  |
| B | 13 | 19 | 19 | 25 | 25 | 29 | 29 | 34 | 34 | 44 |  |
| A | 19 | 27 | 27 | 33 | 33 | 35 | 35 | 38 | 44 | 53 |  |

## CONCUSION

The observations proved that the application of the modified sequencing algorithm to $\mathrm{n} \times 2$ Job machine matrix and Johnson algorithm gives the same optimal solution, but application of the modified sequencing algorithm to $n \times 3, n \times 4, n \times m$ job machine matrix also give the same solution that of the convectional algorithm,but it should be the checking criteria for getting higher optimal solution along with the conventional algorithms because both cases fail in either way because some times in some problems they both may fail. For the $n \times 2$ problem it is not required to go for the next process if the optimum sequence for the both the methods are same.

## REFERENCES

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[^0]:    - Naga Sai Ram.G is currently pursuing undergraduate degree program in mechanical engineering in K.L University, India, PH-+91 8374591376. Email: sairam27@hotmail.com
    - V.S.Haswanth.G is currently pursuing undergraduate degree program in mechanical engineering in K.L University, India, PH-+91 7207077090. Email: author_name@mail.com
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